Planning a Smart Market for Groundwater in Shaanxi, China

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Abstract

Building on recent work in water markets, this paper demonstrates a case study for a sustainable market of groundwater in Tianqiao, a township in Shaanxi Province, China. We used hydrological optimization, with groundwater flow modeling and linear programming, to simulate a smart market, i.e., a market cleared with the help of a computer model. Results show that exchanges would be frequent, as water requirements and availability changed over time, and prices would likely be highest in September, with the rest of the year having quite moderate prices. The proposed mechanism allows a market to be operated weekly, or even daily, with low transaction costs. Users would have useful price information, which would incentivise greater efficiency. The sustainability of the resource is guaranteed by explicit constraints in the optimization.

1 Why a smart market for groundwater is needed

1.1 Ground water allocation is complicated

Most goods and services are easily allocated between pairs of individuals with standard mechanisms of commerce. One person willingly provides a haircut to another person in exchange for money, and other people in the neighborhood are not affected. In contrast, groundwater is shared. Everyone’s use affects everyone else’s use. These effects can take weeks or months to be felt, and accumulate over all uses. As people take more and more water, the environmental effects increase until they are unsustainable. At this point, users can increase water-related production only by improving water efficiency, damaging the environment, or taking water from someone else.

Water trading occurs regularly in some areas, especially where water is controlled with reservoirs, aqueducts, and races. For example, Australia has regular auctions for reservoir water. .

Users can trade because the catchment regulators can reasonably assure delivery; one user takes a megaliter less from the reservoir, and another user takes a megaliter more, and money changes hands. Even with reservoirs, existing water markets have high transaction costs, because making a deal requires finding a trading partner, negotiating a price, writing and enforcing a contract, and obtaining approval through protracted government procedures. To our knowledge, all the Australian water markets are pair-wise or quite simple auction systems for water from reservoirs, with very simply hydrology. None of those auctions take into account complex hydrological interactions, as occurs with groundwater. WaterMove, for example, finds a single "pool price" for a given trading zone (WaterMove 2009). Their "algorithm" is to simply find the lowest price at which supply equals demand. The auction ignores hydrological effects over time, simplifying the hydrology into crude by-season or by-year blocks. The auctions do not have environmental constraints. And the trading still has large transaction costs, as seen by the term lengths of the purchases – usually a season, year, or permanent.

In New Zealand, a firm called HydroTrader (HydroTrader 2009) has developed a fairly typical auction system: users post an interest in buying or selling ground water, and HydroTrader helps clear an EBay-type of auction, which is a pair-wise trade between two users. The transaction costs and times are formidable – typically about NZ$5,000 and two to three months to arrange a trade, including the necessary government approval. WaterMove is many-to-many, but assumes trivial hydrology; complex hydrology as in the case of ground water must be addressed by government vetting. One to one trading with groundwater is complicated to arrange, because each user has partial effects on many other users, and these effects have considerable delay. If the resource is fully allocated, and one person wants more groundwater next week, several other people in the catchment must give up water now, because the effects of the first person's withdrawals are distributed over time. Withdrawals now compete with future withdrawals. As a result of these coordination issues, every trade requires a lengthy government approval. In addition to coordination between users, society would like to ensure that the groundwater use is sustainable. Sustainability can be defined as living within a set of constraints which prevent society from depleting shared resources. This specification of sustainability can take simple forms, e.g., that the aquifer level at each of a set of control points may be drawn down no more than a certain amount, that streams have minimum flows, and that salt-water does not intrude into coastal aquifers. A trading scheme with such limits may be viewed as a "cap and trade," as for carbon emissions. But cap and trade for carbon has just one global limit, and we would like to impose a number of caps, perhaps one for each stream or lake, several for each aquifer, and for every time period. Many hydrologists recognize that problems of this type may be solved by hydrological optimization, but the trading scheme is new.

We would like a system that enables water users to buy or sell water rights easily, where exchange is not one-to-one, but many-to-many. We would like the system to ensure that trade is sustainable, so trades now do not cause environmental problems now or in the future. This detailed level of coordination seems virtually impossible, but this coordination may be done using a *smart market*. Indeed, users could trade every day if they wished.

McCabe, Rassenti and Smith (1991) observed that difficult markets can be assisted by a computer model. For example, electricity generators who add power to a large grid cannot determine which user takes that power. Generators and users simultaneously add and withdraw power. Power must balance at each point, and line capacities cannot be exceeded. Now, electricity markets operate routinely, and these markets are cleared with the help of optimization models that balance the simultaneous power flows (Hogan, Read and Ring, 1996; Alvey et al, 1998). Similarly, natural gas is injected or withdrawn from a national pipeline. The gas may be traded through a computerized market that simulates the gas flow in the pipeline. These markets are called "smart" because they are enabled by underlying computer models, which simulate the physical processes, and then allocate rights between users in a way that maximizes total welfare, subject to the physical constraints. Clearing such markets would be impossible pairwise, or via an ordinary auction as for artwork or used cars; those simple types of exchange cannot manage the physical interactions or the required coordination.

To our knowledge, no water markets anywhere in the world are cleared with the use of optimization. All are cleared either pairwise, or by finding a single price which would match supply to demand within a homogeneous or nearly-homogeneous pool. These markets suffer from high transaction costs (though less so for the many-to-many auctions), and lack explicit constraints for the environment. Most operate over simply hydrology. The proposal here is a much richer system, which correctly balances water across complex hydrology, with explicit constraints for sustainability, and near-zero transaction costs.

Murphy et al (2000) first proposed a smart market for water. Their model was based on a highly simplified model of the southern California surface water network, with only a few pipe segments, for a single period, to conduct economics experiments. They found that the market tended to converge quickly to efficient outcomes.

Our smart market for groundwater in Tianqiao, Shaanxi, follows Raffensperger & Milke (2005) and Raffensperger, Milke & Read (2008), and is a full case study of the associated technical requirements, from a hydrologist's point of view. This paper explains how the market works, but the focus is on Tianqiao.

The smart market for water works quite differently to any market which most people commonly use. Trading is not pairwise as with haircuts, but through a centrally managed pool. The auction is not cleared in a simple highest-bid-wins manner, but rather with the help of a linear program. The linear program is fairly standard hydrological optimization; the objective coefficients come from users' bids, which could be submitted to the auction via a web page. In this way, users need not search for trading partners. They just use the web page. They need not bargain, but simply bid. They do not need to write or enforce contracts, because the market manager enforces market rules. The government approval is automated for the entire catchment within a few minutes, when linear program runs. The linear program ensures that all head constraints are respected. Therefore, the transaction costs go nearly to zero, and users could trade on a weekly, daily, or even hourly basis.

1.2 How the smart market works

The market would be set up for a single catchment. The water authority must choose a set of environmental standards for the catchment, such as the allowed drawdown in the aquifer at each of a set of control points, and must have an adequate hydrological simulation of the catchment.

To set up the market, users' rights must be legally tradable on a short-term (lease) basis. The right is defined in appropriate units, such as cubic meters per day or liters per second, taken from the user's validated well. The length of the right would correspond to the time between auctions. (However, see Raffensperger (2009) for other possible approaches for specifying rights.) Users must be certified to enter the market. Their wells must be metered with appropriate accuracy. Users must follow the market regulations. They must show that they are able to pay when required to do so.

Some countries such as New Zealand have a "use it or lose it" rule, where users are granted rights to take water up to some limit L; if they need $N < L$, the government may reduce L, so users are perversely incentivised to waste $L - N$. such rules must be abolished before the market can work, otherwise users will fear that if they lease out a quantity for a short time, then government will reduce their permanent rights.

Bidders should easily be able to avoid selling if they wish, simply by setting a sufficiently small price for quantities over their current position. Similarly, bidders could avoid buying by setting a sufficiently high price for quantities at and below their current position. Because the right is an option to take water, users may be tempted to waste water in a given auction period, when they have purchased rights for more than they need. A solution for this is to have an ex post auction, in which users are compensated (at later and perhaps discounted prices) for unused water.

As the largest water users have the greatest environmental impact and the highest need for water, it is reasonable that a market could be set up first with the largest users. Smaller users could be admitted to the market as they wish.

The water authority must designate a market manager, who would be responsible for the market's operation. The market is a periodic auction. The auction could be run as often as the users wish, but once per week during the growing season would be reasonable. Market operation is as follows:

- The market manager accepts users' bids for water, whether to buy or sell. The bidding process could be done in person, by phone, by letter, or over the internet.
- The market manager then runs a hydrological optimization, which is a combination of flow modeling and linear programming, to reallocate water among all users simultaneously. This reallocation will tend to give more water to those who bid highest, but constrained by the set of environmental requirements.
- The market manager then announces the results to the users.
- If desired, several preliminary auctions may be run in order for users to develop expectations for prices. However, trading rules should constrain user's bid changes to prevent gaming.

1.3 Benefits of a smart market

Users, regulators, and the public would enjoy many benefits from using a smart market for groundwater allocation.

- Users would be able to change their allocations frequently, perhaps weekly if not daily. This is much more frequent than existing water market schemes, in which water is traded typically once per season. The optimization maximizes the total welfare.
- Users would face the correct price for water, taking into account all other possible uses, and the effects on the environment. Users who face a high price would enjoy sales. Users who face a low price would enjoy being able to obtain more water for productive use.
- The system would be voluntary. Users who do not wish to participate still get their existing rights, but would have to participate to obtain water in excess of their existing rights. However, everyone's rights would need to be adjusted to satisfy the current weather conditions.
- Regulators would get a "bird's eye" view of the catchment, with easier management of users' allocations, and greater certainty of the effects of users' abstractions. Regulators could control environmental flows much more easily.

- The public would benefit from more reliable environmental flows, as these flows would be checked by computer at every auction.

2 The hydrological optimization

2.1 Study site

The study site, Tianqiao, is located at the northeast of Shaanxi Province, with the Yellow River going through it (Figure 1). The total area is about 162.9 km^2. Tianqiao is a small part of the Shanxi-Shaanxi Loess Plateau, and the elevation there ranges from 820 to 1,000 meters above the mean sea level. The climate of Tianqiao is continental and semiarid. Tianqiao receives an average of 440 millimeters of rainfall per year, with most precipitation during June, August and September. In this area, the wind usually comes from the northwest during spring and winter, and from the southwest during summer and fall, with the greatest speed of 24 meters per second.

The main aquifer in Tianqiao consists of carbonate rocks ranging in age from Cambrian to Mid-Ordovician. The Cambrian stratum is mainly composed of limestone and dolomitic limestone; the Lower Ordovician stratum primarily contains dolomite and dolomitic limestone; the Mid-Ordovician stratum consists mainly of limestone, dolomitic limestone and dolomitic marlite. Following the different lithology among these three strata, the aquifer can be divided into three sub-aquifer zones: the Cambrian, the Lower Ordovician, and the Mid-Ordovician. Although marlite or shale may be within the aquifer-forming confining beds, those sub-aquifer zones still connect hydraulically, since the confining beds are thin and their distribution is spatially discontinuous.

This aquifer is comprised of carbonate rocks, so the major type of water contained in it is karstic fracture water which is alkalescent and rich in HCO_3^-, Ca^{2+} and Mg^{2+} (the meq percentage for each of these three ions is larger than 25%). The storage of karstic fracture water in the aquifer is dominated by the faults and folds. In the region between Qingshuichuan and Tianqiao where the faults and folds develop, water yield ranges from 15,000 to 30,000 m^3/day per well. However, the rest of the area has been seldom affected by tectonic activities, so water yield is no more than 200 m^3/day at each of most wells.

2.2 The flow model

2.2.1 The governing equation

Based on the geology and hydrogeology, we can conceptualize this aquifer as a three-dimensional, heterogeneous and anisotropic flow system under transient conditions, with the following governing equation:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y \frac{\partial H}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z \frac{\partial H}{\partial z}\right)-W=S_s \frac{\partial H}{\partial t} \tag{1}$$

where

K_x, K_y, K_z = the principal components of hydraulic conductivity along the *x, y, z* coordinate axes (LT^{-1}), assumed to align with the three orthogonal principal directions of hydraulic conductivity.

H = hydraulic head (L).

W = volumetric flow rate of fluid sinks/sources per unit volume of aquifer (T^{-1}).

S_s is the specific storage (L^{-1}); t is the time (T).

Equation 1 with the initial and boundary conditions constitute the numerical groundwater flow simulation. We used MODFLOW (Harbaugh et al., 2000), a three dimensional finite-difference groundwater flow model, to solve Equation 1 for hydraulic heads and flow rates.

2.2.2 Spatial and temporal discretization

To set up a finite-difference groundwater flow model for simulating the groundwater flow system in Tianqiao, we discretized the aquifer as a three-dimensional grid with three layers representing the three sub-aquifer zones. The plan view has two kinds of discretization. In the Tianqiao Water Supply Source, the gray area in the middle of Figure 2, cells have a constant spacing of 20 m. The remainder is represented by cells with a constant spacing of 100 m (Figure 2). Thus, the grid consists of 3 layers, 279 rows and 190 columns, resulting in 159,030 active cells. Based on the data available, the simulation horizon is the year from September 1, 2000 to August 31, 2001. We used 12 stress periods, with each stress period representing one month. For the market model, however, we used stress periods in weeks.

2.2.3 Boundary and initial conditions

The boundary conditions were specified head at the eastern, northern and southern edges of the study area. The specified head values on those boundaries are calculated from a regional groundwater flow model set up by Shao et al. (2007) for a larger area covering the study site and its surrounding places. The western edge, serving as a groundwater divide, was defined as a no-flow boundary since it also serves as the eastern edge of a stagnant area to the west of the site. Additionally, a general head-dependent boundary was imposed along the Yellow River within the model area. The hydraulic conductance of the interface between the aquifer cells and the Yellow River was estimated at the start of simulation and adjusted during model calibration.

The model's initial condition requires specification of the hydraulic heads at active model cells at the start of the simulation. For this study, the initial hydraulic heads were first determined by using the regional groundwater flow model mentioned above, and then adjusted according to the observed heads at the end of August 2000.

2.2.4 Sources and sinks

Net recharge to the water table is a function of precipitation, infiltration and evapotranspiration, and is applied to the uppermost active layer. Net recharge rates are estimated and assigned based on regional recharge rates given by Shao et al. (2007), and then adjusted during calibration. The model area has several pumping wells (Figure 3), specified with the MODFLOW Well package. These wells extract ground water from the Mid-Ordovician sub-aquifer zone, represented by layer 1 in the model.

2.2.5 Assignment of hydrogeologic parameters and model calibration

Based on the hydrogeologic conditions, we estimated initial zonation and parameter values for hydraulic conductivities, specific storage, and specific yield. Zonation and parameter values were then adjusted during the calibration. (More detailed information is provided by Shao et al., 2007.) We calibrated the model through trial-and-error by adjusting zonation and values of hydraulic conductivities, specific storage, specific yield, and net recharge rates. The observed head values included two observed throughout simulation and 22 one-time observation points monitored at the end of simulation (Figure 3) These four parameters were adjusted until the MODFLOW-calculated heads matched the observed heads to a satisfactory degree (Figure 4).

2.2.6 Wells, control points, and stress periods

For the smart market model, we added 29 potential pumping wells and 10 head control points (Figure 5). Initial heads for the smart market model came from the heads for August 31, 2001, as calculated by the calibrated MODFLOW model. The simulation time for the smart market model (optimization period) was the 78 weeks starting from September 1, 2001. Therefore, the smart market model used 78 stress periods, each representing one week. The 29 potential pumping wells worked from period 1 to 52, stopping after stress period 52; the control point constraints were active for all 78 weeks. The extra 26 weeks were included out of concern that abstraction in the first 52 weeks might have a delayed effect into the following weeks. This proved not to be the case.

2.3 The linear program

We assume one well per user. Should a user own more than one well, the user would have to submit separate bids for each well. A well may be owned and operated by a representative body, such as a city, in which case the city manager would submit bids on behalf of the city.

Each bid consists of a reservation price P^t_{wb} for water and a demand quantity Q^t_{wb} for which that value holds. As with any resource, usually the first quantity of water has highest value. As more water is taken, each successive quantity has lower marginal value. This was modeled in the linear program using six bids, with each bid corresponding to one tranche of a step function.

Indices

b, demand tranche, b=1,…, B. Each user's demand function is modeled in piece-wise linear fashion with B tranches.

w, wells, with w =1,…, W.

k, *l*, control points, with k, l=1, …, K.

u, *t*, time periods, with u, t=1,…, T. Period 1 is the present period.

Parameters

C_w^t = initial quota at well w, period t., in m^3/week. It is the initial quantity position in the market of user w for period t. This quota may have been granted free or purchased earlier. It may be a combination of "permanent" and leased rights.

Q^t_{wb} = the bound on marginal demand tranche b for abstraction at well w, period t.

P^t_{wb} = the reservation price for demand quantity Q^t_{wb} at well w, period t.

F^t_{wk} = draw down response after t periods at control point k, due to abstraction in the current period at well w, in meters of water per cubic meter of water per second. Note that F^1_{wk} is the effect in the current period from abstraction in the current period.

L_k^t = the lower bound on cumulative drawdown at control point k, period t, in meters.

U_k^t = the upper bound on cumulative drawdown at control point k, period t, in meters.

Decision variables

$demand^t_{wb}$ = quantity at value P^t_{wb} accepted for the user at well w, period t, tranche b.

buy_w^t, $sell_w^t$, = amount that the user at well w buys or sells in period t.

p_w^t = market price per unit of water at well w, period t. This is the dual price on constraint 4 below.

λ_k^t = shadow price per unit head at control point k, period t. This is the dual price on constraint 6 below.

q_w^t = abstraction from well w during period t.

The wells in the Tianqiao case were considered capacitated. However, an individual user's well capacity is the responsibility of that user, and has no bearing on the market. A user could (perhaps foolishly) buy more rights than he or she could physically pump. In any case, we viewed this nominal capacity as the initial right C_w^t in the market, so users could buy additional rights, or sell their given rights.

Maximize $\sum_{w=1}^{W}\sum_{t=1}^{T}\sum_{b=1}^{B} P_{w,b}^{t} demand_{w,b}^{t}$ subject to (2)

$0 \le demand_{wb}^{t} \le Q_{wb}^{t}$ for tranches b=1,…, B, wells w=1,…, W, and periods t=1,…, T. (3)

$q_w^t = \sum_{b=1}^{B} demand_{w,b}^{t}$, for demand tranches b=1,…, B, wells w=1,…, W, and periods t=1,…, T. (Dual variable p_w^t.) (4)

$q_w^t = buy_w^t - sell_w^t + C_w^t$, for all wells w=1,…, W, and periods t=1,…, T. (5)

$\sum_{w=1}^{W}\sum_{u=1}^{t} F_{w,k}^{t-u+1} q_w^u \le U_k^t$, for all control points k=1,…, K, and periods t=1,…, T. (Dual variable λ_k^t.) (6)

Explanation:

Equation 2. The objective function maximizes the total value of water. The coefficients P_{wb}^t come from users' bid prices.

Equation 3. Demands are bounded by their marginal quantities. The coefficients Q_{wb}^t come from users' bid quantities.

Equation 4. The well abstraction equals the total demand. The dual variable p_w^t is the marginal value to the market for another unit of water at well w, period t.

Equation 5. The well abstraction equals what is bought, less what is sold, plus the initial licensed rate. The coefficient C_w^t is the initial quantity held by user w in time t. This constraint need not be part of the linear program, but can be calculated after the model solves.

Equation 6. Cumulative drawdown at each control point may have lower and upper bounds. The coefficients L_k^t and U_k^t are set by the water authority as the caps for each environmental control point.

2.4 Bids

The bids are notional, and we assumed that all users have similar value for water, as a % of total water. Table 1 shows the six bids corresponding to the value function in Case 1. These bids appear in the objective function of the model, and "drive" the model. The nominal capacity R_w of each well w (shown in Table 2) was used for C_w^t.

3 Results

We examine three cases. Hydrological engineers often wish to maximize total abstraction. This serves as an initial Case 0 to indicate the maximum sustainable water in the catchment, and it is interesting to consider the economic value associated with this case. In Cases 1, we maximize economic value first subject to well capacity constraints, then in Case 2 without a capacity constraints. The demand curves are the same in cases 1 and 2; the capacity constraints alone are different. The goal is to examine what would happen if capacity constraints were removed, while still ensuring sustainability. This would be the case if, for example, new wells were added near existing wells, or existing wells chose to upgrade. We examine the trade-off between total water taken, capacity, and total economic value.

3.1 Case 0 (baseline): Maximize total water that can be taken sustainably.

In this case, users were assumed to have equal value for water, at every well and at every time period. This serves simply to maximize the total water that users can take sustainably, that is, without violating any constraints at the control points. The only caps on pumping were implicit, as the 10 control point constraints.

A total of 165,692,628.9 m^3 of water could be sustainably extracted from all wells over the year, about 43% more water was extracted than in Case 1, as we shall see. In the solution (shown in Table 2), wells 8, 10 through 15, and 22 were closed down.

As stated, total economic value was not maximized in this Case 0. Rather, the total amount of water was maximized. However, we calculated the economic value associated with this solution to be ¥20,711,600, an order of magnitude lower than the economic value we will obtain from Cases 1 and 2. The solution that maximizes total water does not provide a solution that maximizes economic value.

3.2 Case 1: Maximize economic value, with capacity at twice nominal.

In this case, bids were as shown in Table 1 , with demand in each tranche 1 to 6 of (nominal capacity)/3. Thus, the true total well capacity is twice the nominal capacity. While we have data on pump capacities, these capacities should be viewed as private information from the market point of view. A grocery store does not care if we buy more groceries than will fit in the car. Further, users would probably change their pump capacities over time to match the economic optimum. The price of water differs over time and location, because price depends on the hydrology, on when the water is taken, and where, even when we assume that all users have the same value for water in every period.

In the result, most wells were limited in some time period by this upper bound. However, average abstraction was higher than the nominal capacity only for wells 1 to 4. Total economic value was ¥251,495,421, a factor of 12 greater than Case 0. Total water = 115,548,000, not quite 70% of the total water from Case 0.

3.3 Case 2: Maximize economic value, with no bound on capacity.

In this case, bids were as shown in Table 1, with demand in each tranche 1 to 5 of (nominal capacity)/3, and tranche 6 was not capacitated. However, tranche 6 was low utility of only ¥0.125, to reflect that low marginal value for a high quantity of water. We would expect that the market would move water away from low value bids towards high value bids. As with Case 0, the only caps on pumping were implicit in the 10 control point constraints, but unlike Case 0, the value of the last bid quantity was small for those wells that took it. This case is considered the most realistic scenario for design purposes, because at the design stage, we still have an opportunity to change the pump capacity.

In the result, only well 29 took significantly more water compared to Case 1. Otherwise, Case 2 was quite similar to Case 1. Total economic value ¥252,343,259.5, only slightly higher than Case 1. Total water abstracted was 121,888,000 m^3, about 6% higher than Case 1, demonstrating that the extra water had only small utility.

The next section shows prices for Case 2, for each well and each week.

3.4 Prices for Case 2.

Table 3 shows weekly prices for all wells. Wells are displayed from right to left, in order of decreasing average price. At left, wells 23, 20, and 19 have the most expensive water. At far right, wells 1 to 4 have access to cheaper water. The price of water clearly varies over time, with the expensive period in weeks 37 to 40. The highest price of ¥7.6 occurred at well 5 in week 40, however in most other weeks, well 5 enjoyed relatively lower prices.

3.5 Prices by control point

We expected that the control point constraints would require additional periods beyond the number of periods required for pumping. The reasoning for this was to prevent taking of excess water that had a delayed effect beyond the length of the planning horizon. For this data, however, this was unnecessary, because all control points constraints were loose after the final pumping period.

Figure 6 shows prices by week and control point, omitting those control points and periods that were always priced at zero. Interestingly, constraints associated with control point 5 were always slack. except for period 39, when the constraint was binding at a high price.

3.6 Setting initial rights C_w^t for each well

Initial rights for each well should be set in such a way as to ensure that the central water management authority does not pay users to hold back. The reason for this is to ensure correct incentives in the bidding process, and also to correctly impose the shortage costs onto users.

Consider the case if users are given no rights to take water ($C_w^t = 0$). Users must purchase all water through the market. In this case, the central authority will retain all the money from water purchased.

On the other hand, if users are given rights to take more water than is actually available ($C_w^t >> 0$), the market system will require the central authority to purchase water rights from users, so that total rights are brought down to match the total amount of water. In this case, the central authority will pay money to the users. Depending on the degree that the total rights are above total water, and depending on users' savviness and ability to collude, they could bid up the price of water arbitrarily high. The market manager would have to pay out huge sums to users, which is clearly untenable.

However, rights can be adjusted up or down via the market model so that the money transfers only between users, with no money going to or from the central authority. This could be done in a number of ways, the simplest of which is to choose a proportion α such that the market manager's net revenue $\sum_w p_w^t(\alpha C_w^t - q_w^t) = 0$, that is, $\alpha = (\sum_w p_w^t q_w^t)/(\sum_w p_w^t C_w^t)$, as shown in Raffensperger (2008).

3.7 Limitations

The model did not have head constraints at each well. It is possible that some wells may have had drawdown below their assumed layer.

4 Conclusions

We have demonstrated the technology required for operation of a smart market for groundwater allocation in Tianqiao. This technology includes conversion of a standard hydrological optimization to a model appropriate for clearing a water market.

We have also suggested a range of institutional parameters, rules, and procedures in order for the market to work. The result is a flexible system in which groundwater can be reallocated on a weekly basis to its best use.

The amount of water that can be taken sustainably depends on where and when it is taken, and this should be weighted by the relative value of the water. Our results show that the price of water is driven by rainfall and by distance to binding control points. Wells closer to control points tend to face higher prices, and wells further from control points tend to face lower prices.

We also considered some technical issues associated with the response matrix. We noted that many of the coefficients were so small that they could be ignored, without appreciable loss of precision in the model.

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Table 1. Bid tranches. R_w = nominal capacity for well *w*. Bids were identical for all wells *w* and weeks *t*.

Tranche step b	1	2	3	4	5	6
Reservation price P^t_{wb}	¥4	¥2	¥1	¥0.5	¥0.25	¥0.125
Quantity Q^t_{wb}	$R_i/3$	$R_i/3$	$R_i/3$	$R_i/3$	$R_i/3$	Case 1, $R_i/3$. Case 2, infinity.

Table 2. Quantities q_w^t in the three cases (000 m^3).

		Case 0			Case 1			Case 2		
Well *w*	Nominal capacity C_{wt}	Max	Avg	% avg /nominal	Max	Avg	% avg/ nominal	Max	Avg	% avg/ nominal
1	300	874	16.8	6%	516.6	383.7	128%	500	385.2	128%
2	300	1,379.00	1,012.70	338%	600	393.4	131%	646.7	398.4	133%
3	300	1,862.40	470.8	157%	600	355.8	119%	994.8	376.5	125%
4	300	1,412.40	403.6	135%	600	369	123%	1,115.10	373.4	124%
5	50	2,084.30	806.5	1613%	100	47.4	95%	1,435.20	94.6	189%
6	50	1,387.40	102.3	205%	100	48.7	97%	684.7	69.5	139%
7	50	631.9	31.8	64%	100	42	84%	83.3	39.6	79%
8	50	0	0	0%	83.3	33	66%	66.7	31.8	64%
9	50	385.2	19.8	40%	83.3	36.2	72%	66.7	34.5	69%
10	50	0	0	0%	66.7	29.2	58%	66.7	28.5	57%
11	50	0	0	0%	66.7	30	60%	66.7	29.2	58%
12	50	0	0	0%	83.3	34.4	69%	66.7	33	66%
13	50	0	0	0%	71.4	27.7	55%	70.8	27.4	55%
14	50	0	0	0%	83.3	30	60%	50	28.7	57%
15	50	0	0	0%	93.7	36.5	73%	66.7	34.2	68%
16	50	420.5	8.1	16%	83.3	32.9	66%	83.3	31.4	63%
17	38	560.3	26.2	69%	76	20.9	55%	274.9	24	63%
18	38	570.8	16.3	43%	76	24.1	63%	160.4	24.5	65%
19	38	759.9	24.6	65%	76	21.7	57%	63.3	20.9	55%
20	38	443.8	26.6	70%	76	21	55%	286.7	27.7	73%
21	38	109.2	2.1	6%	76	22.5	59%	122.7	22.4	59%
22	38	0	0	0%	63.3	20.4	54%	50.7	20	53%
23	38	494.1	17.4	46%	52.4	17.2	45%	50.7	16.7	44%
24	38	535.9	20.3	54%	76	27.2	72%	346.4	30.8	81%
25	38	294	5.7	15%	76	22.6	59%	63.3	21.7	57%
26	38	529.4	39.6	104%	76	25.1	66%	289.4	28.6	75%
27	38	206.8	4	10%	76	22.5	59%	299.3	26.1	69%
28	38	475.2	13.6	36%	75.6	20.5	54%	63.3	20.6	54%
29	38	795.3	117.8	310%	76	26.5	70%	481.6	44.3	117%
Total			3,186			2,222			2,344	

Table 3. Weekly prices p_w^t for each well *w*, in ¥/m^3.

Cheaper ——— More expensive

Color key	From	¥0	¥2	¥3	¥4
	to	¥2	¥3	¥4	¥8

	← Wells closer to control points																Wells further from control points →											
Week t	23	20	19	21	17	22	28	27	25	18	24	26	13	29	10	14	11	8	16	12	15	9	7	5	6	3	4	1	2
1	2.3	1.8	2.4	2.3	2.2	0.4	0.2	2.3	0.2	2.2	2.4	2.0	2.1	2.4	2.1	2.0	2.0	2.0	2.0	2.0	2.0	2.0	0.4	1.8	0.1	0.4	0.4	0.4	0.4
2	2.1	2.3	2.7	2.3	0.5	2.5	2.5	0.5	0.5	2.6	0.3	2.3	2.3	2.3	0.5	2.0	2.0	2.0	2.0	1.8	0.4	1.8	2.1	2.0	1.9	0.3	0.3	0.3	0.3
3	2.8	0.5	0.4	2.8	2.6	2.4	0.4	2.8	2.6	2.6	2.6	2.8	1.9	2.8	1.9	1.9	2.2	2.1	2.2	2.1	2.0	2.2	0.3	0.1	1.7	0.6	0.6	0.7	0.7
4	2.9	2.8	2.9	3.0	2.9	0.8	0.8	3.0	3.0	2.6	2.8	3.0	2.0	0.8	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.2	2.2	0.8	0.8	0.8	0.8
5	3.8	3.7	3.2	3.9	3.2	1.6	3.5	3.5	3.5	1.2	1.3	1.0	2.3	3.4	2.0	2.4	2.4	2.2	2.3	2.2	2.0	2.2	2.3	2.0	2.1	1.0	1.0	0.9	0.9
6	2.3	2.4	0.4	2.4	2.2	2.3	0.2	2.3	2.3	2.4	2.3	2.2	2.3	2.6	2.1	2.3	2.0	0.4	2.0	2.0	2.0	0.4	1.9	1.8	2.0	0.5	0.5	0.5	0.5
7	2.7	0.7	2.4	1.0	0.6	2.6	2.7	2.8	0.4	2.8	2.9	0.7	1.0	2.6	0.9	0.9	0.9	2.5	0.8	2.5	2.2	2.4	2.1	1.9	1.9	0.6	0.5	0.5	0.5
8	2.4	2.1	2.5	2.5	2.2	2.1	2.3	2.4	2.4	0.2	2.4	2.5	2.4	2.4	2.4	2.3	2.1	0.6	2.3	0.7	0.5	0.6	0.5	1.7	1.7	0.4	0.6	0.3	0.3
9	2.5	2.2	2.2	2.6	2.2	2.5	2.5	2.1	2.1	2.2	1.6	0.2	2.5	0.1	2.4	0.7	2.2	0.6	2.2	1.9	0.6	2.0	1.9	0.1	0.2	0.3	0.3	0.3	0.3
10	2.2	0.1	2.3	2.1	2.3	2.6	2.1	2.2	2.3	0.2	2.6	2.1	2.3	2.1	2.3	2.2	2.2	0.4	2.2	0.4	0.4	0.4	0.4	0.1	0.2	0.3	0.3	0.1	0.1
11	2.2	2.4	2.2	0.1	0.2	2.1	2.2	2.1	2.1	0.1	0.2	2.4	1.6	2.4	1.6	1.6	1.6	1.6	0.2	1.6	1.6	1.6	1.6	1.8	1.8	0.4	0.4	0.3	0.2
12	2.5	2.6	2.1	2.3	2.1	2.6	2.5	0.1	2.4	2.5	2.4	2.5	1.8	0.1	1.8	1.7	1.8	1.8	1.7	1.8	1.8	1.9	1.8	0.1	0.1	0.4	0.4	0.4	0.4
13	0.5	2.5	2.6	0.4	2.6	0.6	2.2	2.5	2.6	2.7	0.2	2.4	2.0	2.3	2.0	1.9	1.9	1.9	2.0	1.9	1.9	1.9	1.9	1.9	1.9	0.5	0.5	0.3	0.2
14	2.1	2.3	2.2	2.1	2.1	2.5	2.4	0.1	0.1	2.2	0.3	0.1	2.9	2.3	2.8	1.1	1.1	1.0	2.6	1.0	0.9	1.0	0.9	0.3	0.5	0.5	0.5	0.5	0.5
15	2.1	2.1	2.5	2.1	2.6	2.1	2.1	2.3	2.3	2.6	0.1	0.4	1.8	2.2	1.8	1.8	1.8	1.8	1.7	1.8	1.8	1.8	1.8	1.7	0.1	0.4	0.3	0.3	0.3
16	2.3	2.6	2.6	1.8	2.3	0.5	2.7	2.6	2.6	2.3	0.4	2.5	2.0	2.5	2.0	1.9	1.9	1.9	0.6	1.9	1.9	1.9	0.1	0.2	0.2	0.5	0.5	0.5	0.5
17	2.6	0.2	2.6	2.3	2.4	0.5	2.3	2.4	2.2	2.6	0.2	2.4	2.0	2.4	2.0	2.0	2.0	2.0	2.0	0.3	1.9	0.3	1.9	1.9	1.9	0.9	0.9	0.5	0.8
18	2.2	0.2	0.2	2.2	2.4	2.1	2.4	2.3	2.3	0.2	2.3	2.3	2.4	2.1	2.3	2.3	2.3	2.3	2.2	0.5	0.4	0.4	0.4	0.1	0.2	0.3	0.3	0.3	0.3
19	2.3	2.2	2.0	2.3	2.0	2.4	2.2	2.2	2.2	2.0	2.0	0.5	1.4	0.1	1.4	1.4	1.4	1.4	1.4	1.7	1.7	1.7	1.6	1.6	1.6	0.3	0.3	0.1	0.1
20	1.8	1.9	1.8	1.8	1.9	2.3	2.0	1.9	1.9	1.8	1.8	1.9	2.7	2.1	2.6	2.5	2.5	2.4	2.4	2.4	2.4	2.4	2.3	0.1	0.3	0.2	0.2	0.1	0.1
21	2.1	2.1	0.3	0.2	2.5	2.5	2.3	2.2	2.3	2.4	0.1	1.5	0.8	0.2	0.8	2.2	2.2	2.0	0.6	0.5	2.1	0.6	0.5	1.6	0.1	0.4	0.1	0.2	0.2
22	2.1	2.2	2.1	2.5	2.1	1.6	2.5	0.2	0.2	2.1	2.1	0.1	0.7	0.1	0.7	0.6	0.6	0.6	0.6	0.6	0.5	0.6	0.5	1.8	1.9	0.3	0.3	0.2	0.2
23	2.2	2.2	2.6	0.9	2.5	2.3	2.2	2.0	2.0	2.6	0.2	0.1	1.7	0.3	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	0.3	0.3	0.2	0.2
24	2.3	0.1	2.6	2.5	2.4	2.5	2.3	2.1	2.1	2.6	2.6	2.1	1.7	0.6	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.6	1.6	0.1	0.1	0.3	0.3
25	2.5	2.2	2.2	2.3	0.1	0.4	2.2	2.4	2.4	0.2	2.2	2.5	1.0	0.3	1.0	1.0	1.0	0.9	0.8	1.0	0.9	0.9	0.9	0.5	0.6	0.3	0.3	0.2	0.2
26	2.2	0.6	2.2	0.1	2.2	0.3	0.2	2.1	2.1	2.1	0.1	2.2	1.1	2.1	1.0	0.9	0.9	0.9	0.8	0.9	0.8	0.8	0.7	0.3	0.4	0.3	0.3	0.2	0.2
27	2.8	2.9	1.1	0.9	0.7	2.9	2.7	0.7	0.7	0.5	0.7	2.6	0.3	2.6	0.3	1.8	1.8	1.8	0.2	1.8	1.8	0.3	0.7	0.2	0.3	0.1	0.1	0.3	0.3
28	1.5	1.1	2.7	3.0	2.7	1.4	1.0	0.8	0.8	2.7	0.8	0.8	1.9	0.7	1.9	2.0	0.3	0.5	1.6	0.5	0.5	1.9	0.5	0.4	0.4	0.1	0.1	0.3	0.3
29	2.4	2.5	2.2	0.6	2.2	2.3	0.6	0.7	0.7	2.2	2.4	0.7	1.7	0.5	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.8	0.4	0.3	0.3	0.3
30	0.4	2.3	0.3	0.3	0.3	2.5	2.3	2.4	2.4	0.3	2.3	2.3	1.1	2.3	1.0	0.9	0.9	0.8	0.9	0.8	0.8	0.8	0.7	0.3	0.4	0.4	0.3	0.3	0.3
31	3.2	1.4	0.9	1.3	1.2	3.3	1.0	1.0	1.0	1.5	1.1	1.0	2.1	2.9	2.1	2.1	2.2	2.2	2.1	2.1	2.2	2.0	0.5	1.9	1.9	0.4	0.4	0.3	0.3
32	1.5	1.1	2.9	1.0	2.9	0.9	1.0	0.8	0.8	2.9	0.9	2.8	0.6	2.8	2.1	2.1	0.6	0.6	0.6	2.1	2.2	2.1	2.2	0.7	0.4	0.5	0.5	0.4	0.4
33	0.8	2.8	1.0	3.0	0.9	2.9	2.8	2.8	2.8	0.9	2.8	0.8	0.8	0.8	0.8	0.8	0.8	2.5	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.4	0.4	0.4	0.4
34	1.3	3.2	3.4	1.2	3.3	1.6	1.2	3.2	3.2	3.4	3.2	3.2	1.2	3.2	2.7	2.7	2.7	2.7	1.2	2.7	2.7	2.7	2.7	1.1	1.1	0.6	0.6	0.4	0.4
35	3.0	3.0	1.0	1.0	1.0	1.3	1.1	1.3	1.3	1.0	3.0	1.4	1.1	1.4	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	2.5	2.5	0.8	0.8	0.4	0.4
36	1.2	2.8	1.4	1.2	3.3	1.1	3.2	1.2	1.2	3.4	3.2	1.3	1.9	1.6	2.0	1.9	1.9	1.9	1.9	1.9	1.9	1.9	1.8	1.7	1.7	0.9	0.9	0.5	0.5
37	2.5	2.5	4.0	4.7	2.8	2.7	4.0	4.0	4.0	2.8	4.7	3.7	3.9	3.9	4.0	3.1	3.1	3.1	3.8	3.1	3.1	3.1	3.0	2.6	2.6	1.1	1.1	0.5	0.5
38	4.0	3.9	5.0	4.3	4.0	3.8	3.9	4.0	4.0	3.0	4.1	4.0	4.6	3.9	4.7	4.6	4.7	4.7	4.6	4.7	4.6	4.7	4.6	5.0	4.9	1.5	1.5	1.0	0.9
39	4.4	4.4	4.4	2.8	4.4	4.3	4.2	4.2	4.2	4.4	4.3	4.2	6.0	4.3	6.0	6.0	6.1	6.1	6.0	6.1	6.1	6.1	6.1	7.6	6.9	1.4	1.3	1.0	1.0
40	5.0	5.6	3.8	5.5	4.0	4.9	4.2	4.0	4.0	3.8	4.4	3.9	2.1	3.6	2.0	1.9	1.9	1.9	1.9	1.9	1.8	1.8	1.7	1.5	1.7	0.6	0.6	0.6	0.6
41	1.6	1.6	1.7	1.7	1.7	1.7	1.8	1.7	1.7	1.7	1.6	1.7	2.2	1.7	2.1	2.0	2.0	1.9	1.9	2.0	1.9	1.9	1.8	1.5	1.5	1.0	0.9	0.9	0.9
42	1.9	2.0	2.0	2.0	2.0	1.9	2.0	2.0	2.0	2.0	2.1	2.0	2.1	2.0	2.1	2.0	2.1	2.0	2.0	2.0	2.0	2.0	1.9	1.6	1.7	1.3	1.2	1.1	1.1
43	1.9	2.1	1.9	2.0	1.9	2.1	1.9	1.9	1.9	1.9	1.9	1.9	2.3	1.7	2.2	2.1	2.1	2.0	2.0	2.0	2.0	2.0	1.9	1.2	1.4	1.1	1.0	0.9	0.9
44	4.7	5.2	3.7	5.3	3.6	4.7	4.0	3.8	3.9	3.7	3.9	3.7	2.3	3.3	2.2	2.1	2.1	2.0	2.0	2.0	1.9	2.0	1.9	1.2	1.4	0.6	0.6	0.5	0.5
45	0.9	0.9	1.0	0.9	1.0	0.9	0.9	0.9	0.9	1.0	0.9	0.9	1.1	0.9	1.1	1.0	1.1	1.0	1.0	1.0	1.0	1.0	1.0	0.7	0.8	0.3	0.3	0.3	0.3
46	2.7	2.9	2.2	2.9	2.1	2.7	2.3	2.2	2.2	2.2	2.3	2.2	2.1	2.0	2.0	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.6	0.9	1.1	0.4	0.4	0.4	0.4
47	2.5	2.8	1.9	2.8	1.9	2.5	2.1	2.0	2.0	1.9	2.1	1.9	1.3	1.8	1.2	1.1	1.2	1.1	1.1	1.1	1.1	1.1	1.0	0.5	0.7	0.5	0.5	0.5	0.5
48	1.0	1.0	0.9	1.0	0.9	1.0	0.9	0.9	0.9	0.9	0.9	0.9	1.2	0.9	1.2	1.1	1.1	1.0	1.0	1.0	1.0	1.0	0.9	0.5	0.6	0.5	0.5	0.5	0.5
49	2.4	2.7	1.9	2.8	1.9	2.5	2.1	2.0	2.0	1.9	2.2	1.9	1.2	1.8	1.2	1.1	1.1	1.1	1.1	1.1	1.0	1.1	1.0	0.7	0.8	0.5	0.5	0.5	0.5
50	1.1	1.2	1.0	1.2	1.0	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.3	1.0	1.2	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.0	0.6	0.7	0.5	0.5	0.5	0.5
51	2.0	2.2	1.6	2.2	1.6	2.0	1.7	1.6	1.7	1.6	1.7	1.6	2.1	1.5	2.0	1.8	1.9	1.7	1.8	1.7	1.7	1.7	1.5	0.7	0.9	0.6	0.6	0.5	0.5
52	2.0	2.3	1.6	2.2	1.5	2.0	1.7	1.6	1.6	1.5	1.7	1.5	1.4	1.4	1.3	1.2	1.2	1.1	1.1	1.1	1.1	1.1	1.0	0.4	0.6	0.3	0.3	0.3	0.3
Avg	2.3	2.2	2.1	2.1	2.1	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.9	1.9	1.9	1.9	1.8	1.7	1.7	1.7	1.7	1.7	1.5	1.3	1.3	0.5	0.5	0.4	0.4

Table 4. Results of changing the cutoff of response matrix coefficients.

Model	Cutoff	Nonzeroes	Objective value
1	None	806,693	¥252,343,259.5
2	$F^{t}_{wk} < 5*10^{-7}$	805,791	¥252,343,259.5
3	$< 10^{-8}$	500,165	¥250,650,766.2
4	$< 10^{-9}$	405,939	¥249,376,152.9
5	$< 10^{-10}$	380,853	¥249,254,142.8
6	≤ 0	377,059	¥249,252,663.1
7	$< -10^{-10}$	373,034	¥249,253,545.1
8	$< -10^{-9}$	345,093	¥249,334,803.2
9	$< -10^{-8}$	234,230	¥250,978,196.7
10	$< -1.25*10^{-8}$	220,471	¥251,248,456.4
11	$< -1.5*10^{-8}$	207,266	¥254,099,826.4
12	$< -2*10^{-8}$	186,168	¥259,811,934.8
13	$< -5*10^{-8}$	115,166	¥267,619,674.0
14	$< -10^{-7}$	73,705	¥273,661,819.2

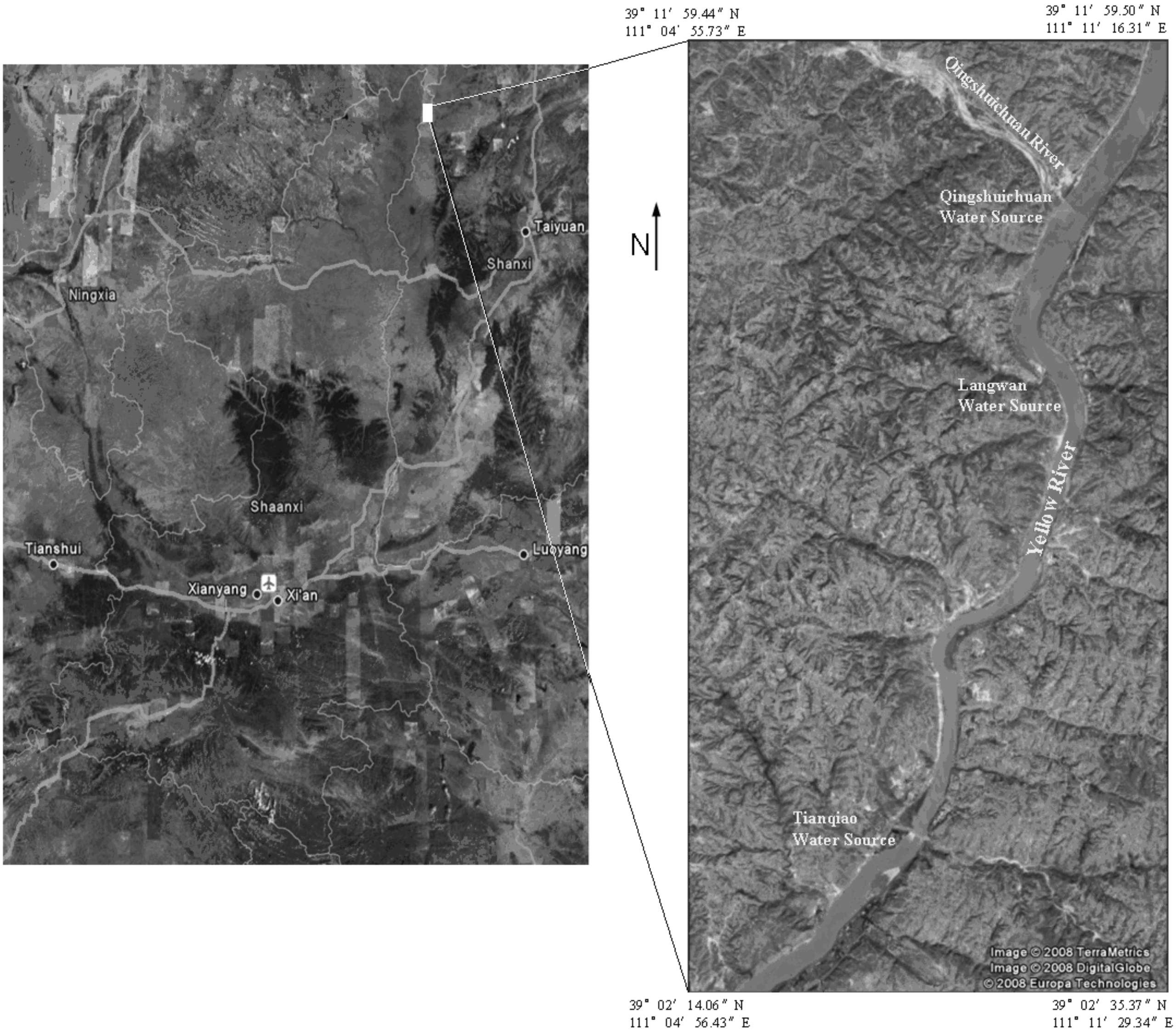


Figure 1. Location and geographical extent of the study area. Modified from Google Earth.

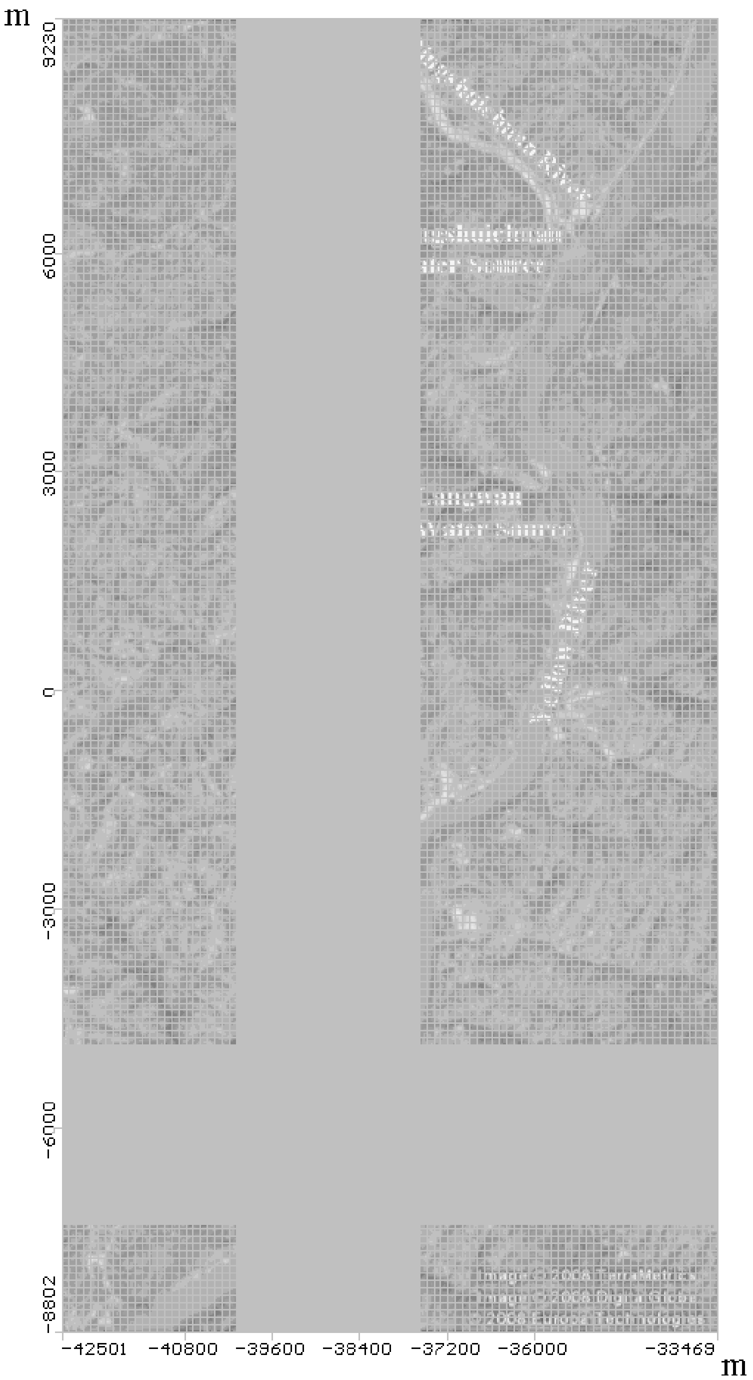


Figure 2. Plan view of the finite-difference model grid. The gray strips indicate the model area with a small uniform grid spacing of 20 m for higher resolution around the Tianqiao Water Supply Source. Map source: Google Maps.

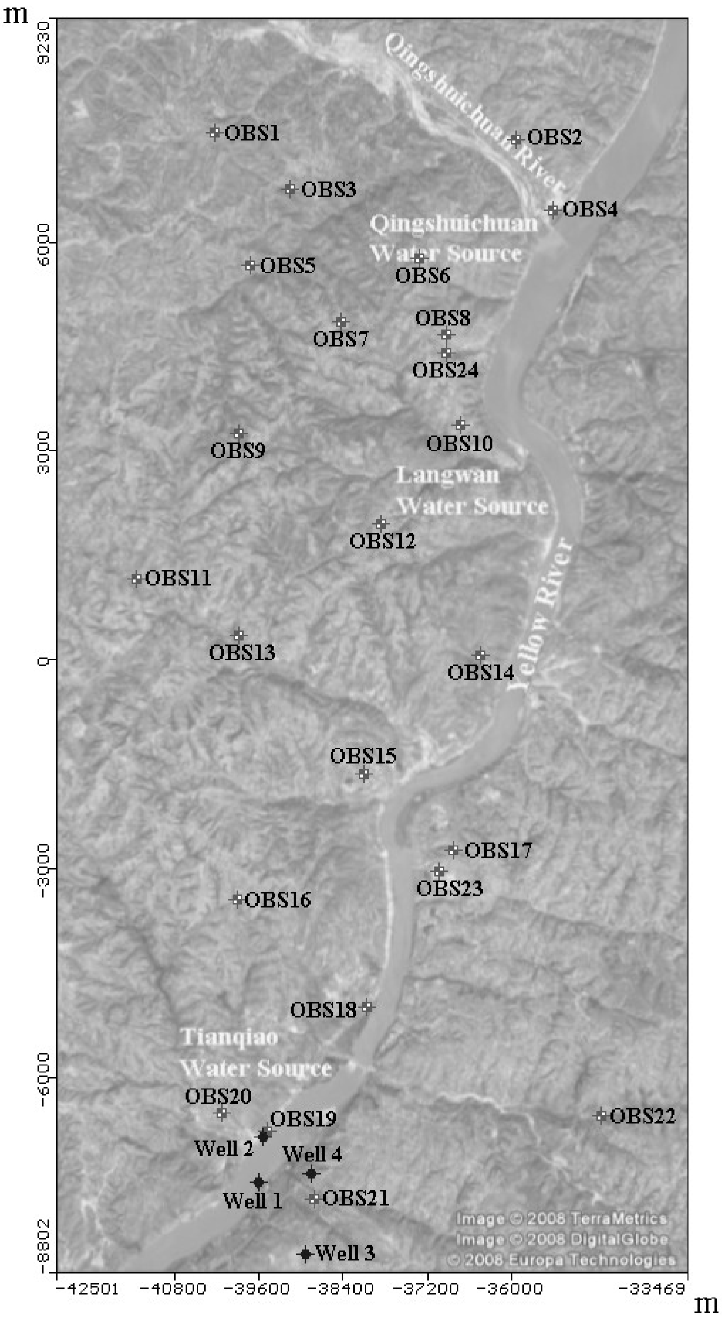


Figure 3. Location of pumping wells and head observation points in the study site. Wells 1 – 4 are pumping wells. OBS denotes head observation points; OBS1-22 are all one-time observation points at the end of simulation; OBS23 and OBS24 are continuous observation points. Map source: Google Maps.

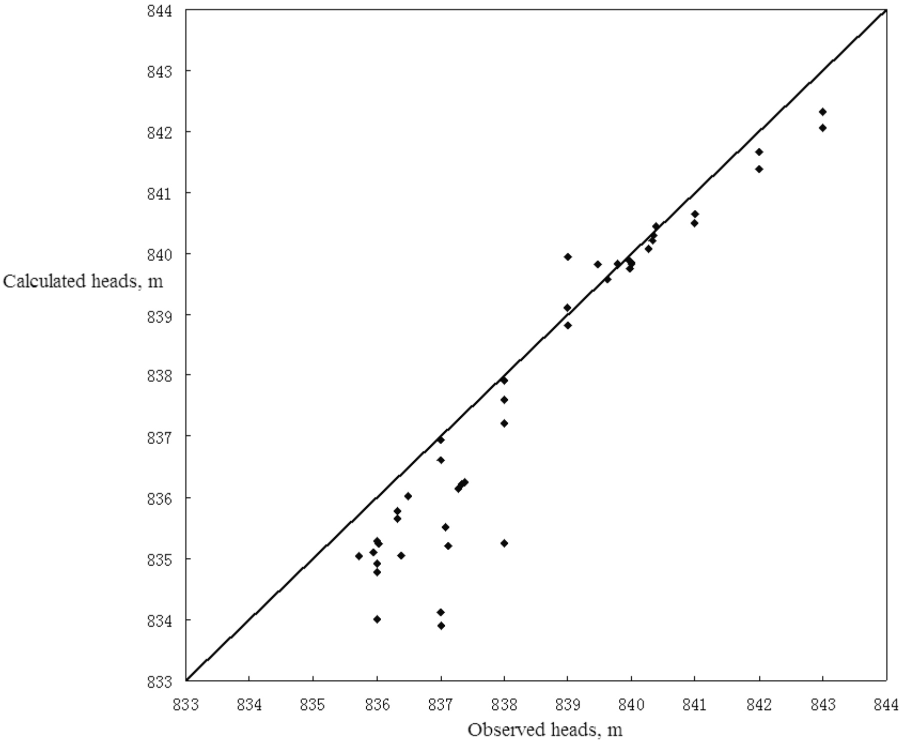
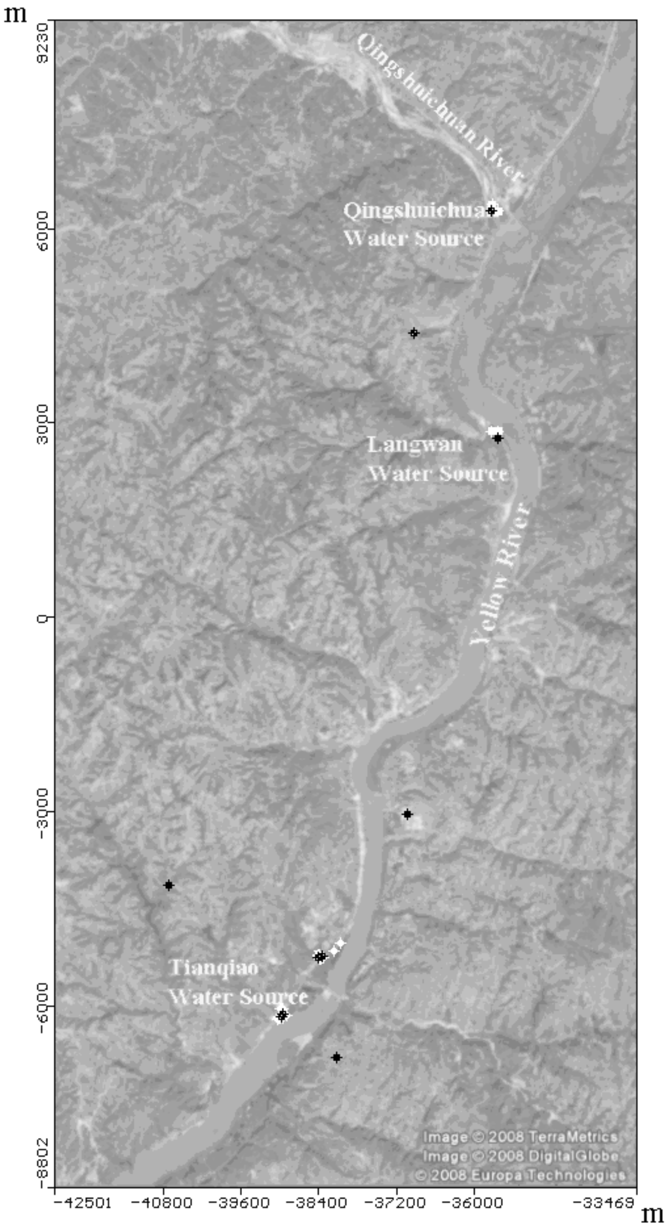


Figure 4. Scatter diagram between the observed and calculated heads at the end of simulation. Model calibration results in a correlation coefficient of 0.954, mean error of -0.696 m, and root mean square error of 1.164 m.



Location of potential pumping wells in the model

Well	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Row	29	30	64	64	150	155	161	159	159	160	160	160	161	161	161	162	200	200	200	204	204	205	205	205	206	207	207	207	208
Col	168	169	168	169	125	120	112	108	109	108	109	110	109	110	111	110	79	80	81	80	82	80	82	84	78	78	79	81	77

Location of head control points in the model

Head control point	1	2	3	4	5	6	7	8	9	10
Row	206	160	238	134	123	49	161	204	30	65
Column	80	111	122	18	155	156	108	81	168	169

Figure 5. Location of potential pumping wells and head control points in the smart market model. White points represent potential pumping wells. Black points represent head control points. Source: Google Maps.

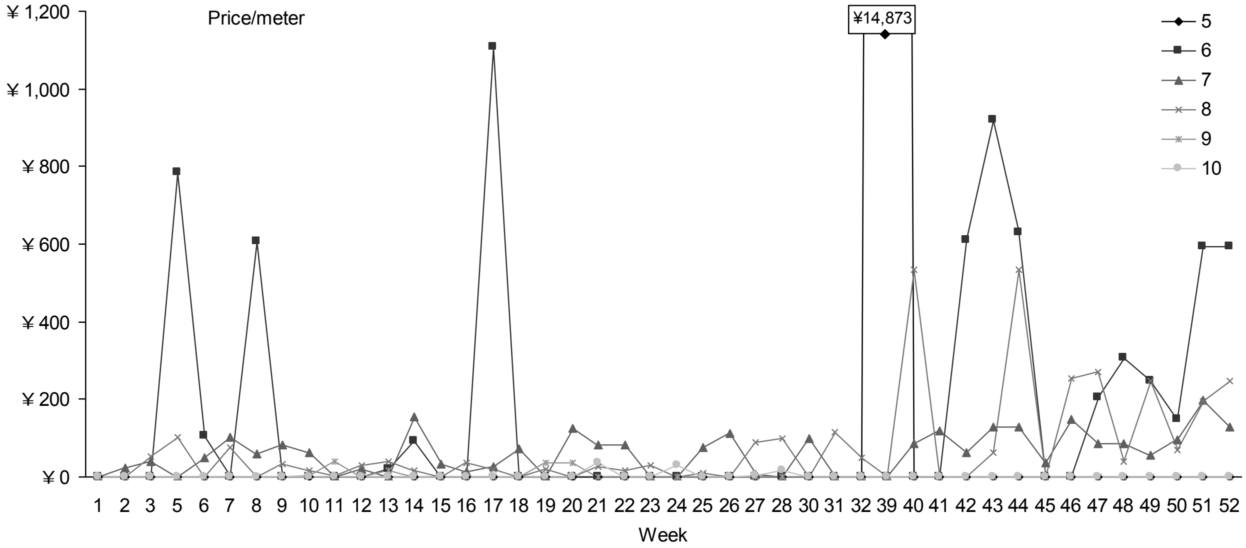


Figure 6. Weekly prices λ_k^t by control point *k*, in ¥/m. Prices for control points 1-4 were ¥0 in all periods. Prices for periods 53 to 78 were ¥0 for all control points.